J.K. SHAH CLASSES

MATHEMATICS & STATISTICS

FYJC TERMINAL TEST - 03

SOLUTION SET

DURATION - 2 HR MARKS - 50

Q1. (A) Attempt ANY **THREE OF** THE FOLLOWING (09)

Q-1A

01. Find k if the equations are consistent

$ky = x + 1$; $2x - 3y + 5 = 0$, $3x + 2y + 1 = 0$

SOLUTION

$x - ky = -1$

$2x - 3y = -5$

$3x + 2y = -1$ are consistent

Hence

$$\begin{vmatrix} 1 & -k & -1 \\ 2 & -3 & -5 \\ 3 & 2 & -1 \end{vmatrix} = 0$$

Taking '–' common from C2 & C3

$$\begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{vmatrix} 1 & k & 1 \\ 2 & 3 & 5 \\ 3 & -2 & 1 \end{vmatrix} = 0 \end{array}$$

$1(3 + 10) - k(2 - 15) + 1(-4 - 9) = 0$

$13 + 13k - 13 = 0$

$13k = 0$

$k = 0$

02. If A.M. of two numbers exceeds their G.M. by 1 and their H.M. by $^9/_5$, find the numbers

SOLUTION

$A - G = 1 \quad \therefore \; G = A - 1 \; ..... \quad (1)$

$A - H = {}^9/_5 \quad \therefore \; H = A - {}^9/_5 \; .... \quad (2)$

$G^2 = AH$

$(A - 1)^2 = A(A - {}^9/_5)$

$A^2 - 2A + 1 = \dfrac{A^2 - 9A}{5}$

$5A^2 - 10A + 5 = 5A^2 - 9A$

$A = 5$

SUBS IN (1)

$G = 5 - 1 = 4$

NOW ;

$A = 5$	$G = 4$
$\dfrac{a + b}{2} = 5$	$G^2 = 16$
$a + b = 10$ ..... (3)	$ab = 16$ .... (4)

Solving

$a(10 - a) = 16$

$10a - a^2 = 16$

$a^2 - 10a + 16 = 0$

$a = 8 \; , \; a = 2$

$b = 2 \; , \; b = 8$. The numbers are 2 & 8 .

03. Find sum of $\frac{1^3}{1} + \frac{1^3 + 2^3}{1 + 2} + \frac{1^3 + 2^3 + 3^3}{1 + 2 + 3} + \ldots..$

SOLUTION

$$tn = \frac{1^3 + 2^3 + 3^3 + \ldots.. + n^3}{1 + 2 + 3 + \ldots\ldots + n}$$

$$= \frac{\frac{n^2(n + 1)^2}{4}}{\frac{n(n + 1)}{2}}$$

$$= \frac{n(n + 1)}{2}$$

$$= \sum_{r = 1}^{n} \frac{r(r + 1)}{2}$$

$$= \sum \frac{r^2 + r}{2}$$

$$= \frac{1}{2}\left(\Sigma r^2 + \Sigma r\right)$$

$$= \frac{1}{2}\left(\frac{n(n + 1)(2n + 1)}{6} + \frac{n(n + 1)}{2}\right)$$

$$= \frac{n(n + 1)}{4}\left(\frac{2n + 1}{3} + 1\right)$$

$$= \frac{n(n + 1)}{4} \quad \frac{2n + 1 + 3}{3}$$

$$= \frac{n(n + 1)(2n + 4)}{12}$$

$$= \frac{n(n + 1)(n + 2)}{6}$$

04. Find square root of : $2(1 - \sqrt{3}i)$

$$\sqrt{2 - 2\sqrt{3}i} = a + bi$$

$$2 - 2\sqrt{3}i = (a + bi)^2$$

$$2 - 2\sqrt{3}i = a^2 + 2abi + b^2i^2$$

$$2 - 2\sqrt{3}i = a^2 + 2abi - b^2$$

$$2 - 2\sqrt{3}i = a^2 - b^2 + 2abi$$

COMPARING

$a^2 - b^2 = 2$ .... (1)

$2ab = -2\sqrt{3}$

$ab = -\sqrt{3}$

$b = \dfrac{-\sqrt{3}}{a}$

subs in (1)

$$a^2 - \left(\frac{-\sqrt{3}}{a}\right)^2 = 2$$

$$a^2 - \frac{3}{a^2} = 2$$

$$\frac{a^4 - 3}{a^2} = 2$$

$$a^4 - 3 = 2a^2$$

$$a^4 - 2a^2 - 3 = 0$$

$$(a^2 - 3)(a^2 + 1) = 0$$

$$a^2 = 3$$

$$a = \pm \sqrt{3}$$

NOW

$a = \sqrt{3}$

$b = \dfrac{-\sqrt{3}}{a}$

$= \dfrac{-\sqrt{3}}{\sqrt{3}}$

$= -1$

HENCE

$\sqrt{2 - 2\sqrt{3}i}$

$= \sqrt{3} - i$

$a = -\sqrt{3}$

$b = \dfrac{-\sqrt{3}}{a}$

$= \dfrac{-\sqrt{3}}{-\sqrt{3}}$

$= 1$

HENCE

$\sqrt{2 - 2\sqrt{3}i}$

$= -\sqrt{3} + i$

$\pm(\sqrt{3} - \sqrt{i})$

(B) Attempt ANY ONE OF THE FOLLOWING (04)

Q-1B

01. Prove without expanding as far as possible

$$\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$$

SOLUTION

$C_1 - C_2 \quad , \quad C_2 - C_3$

$$= \begin{vmatrix} -a-b-c & 0 & 2a \\ b+c+a & -b-c-a & 2b \\ 0 & c+a+b & c-a-b \end{vmatrix}$$

Taking (a+b+c) common from R_1 & R_2 respectively .

$$= (a+b+c)^2 \begin{vmatrix} -1 & 0 & 2a \\ 1 & -1 & 2b \\ 0 & 1 & c-a-b \end{vmatrix}$$

Expanding the determinant

$$= (a+b+c)^2 \Big(-1(-c+a+b-2b) + 2a(1+0) \Big)$$

$$= (a+b+c)^2(c-a-b+2b+2a)$$

$$= (a+b+c)^2(a+b+c)$$

$$= (a+b+c)^3$$

02. Prove without expansion

$$\begin{vmatrix} 11 & 4 & 10 \\ 2 & 7 & 6 \\ 5 & 1 & 4 \end{vmatrix} = 3\begin{vmatrix} 5 & 2 & 3 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$$

SOLUTION

$$\text{LHS} = \begin{vmatrix} 11 & 4 & 10 \\ 2 & 7 & 6 \\ 5 & 1 & 4 \end{vmatrix}$$

$R \leftrightarrow C$

$$= \begin{vmatrix} 11 & 2 & 5 \\ 4 & 7 & 1 \\ 10 & 6 & 4 \end{vmatrix}$$

$C_2 \leftrightarrow C_3$

$$= -\begin{vmatrix} 11 & 5 & 2 \\ 4 & 1 & 7 \\ 10 & 4 & 6 \end{vmatrix}$$

$R_1 \leftrightarrow R_2$

$$= +\begin{vmatrix} 4 & 1 & 7 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$$

$R_1 + R_2$

$$= +\begin{vmatrix} 15 & 6 & 9 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix}$$

Taking '3' common from R_1

$$= 3\begin{vmatrix} 5 & 2 & 3 \\ 11 & 5 & 2 \\ 10 & 4 & 6 \end{vmatrix} = \text{RHS}$$

Q2. (A) Attempt ANY **TWO OF** THE FOLLOWING (06)

Q-2A

01. find the equation of the line so that the line segment intercepted between the axes is divided by the point P(–5,4) internally in the ratio 1:2

SOLUTION

Let the line cut x – axis at A(a,0) and y – axis at B(0,b)

P divides AB internally in the ratio 1:2

A (a,0) —1— P (–5,4) —2— B (0,b)

$$-5 = \frac{1(0) + 2(a)}{1 + 2} \qquad 4 = \frac{1(b) + 2(0)}{1 + 2}$$

$$-15 = 2a \qquad 12 = b$$

$$a = -15/2 \qquad b = 12$$

Equation of the line ; $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{-15/2} + \frac{y}{12} = 1$$

$$\frac{-2x}{15} + \frac{y}{12} = 1$$

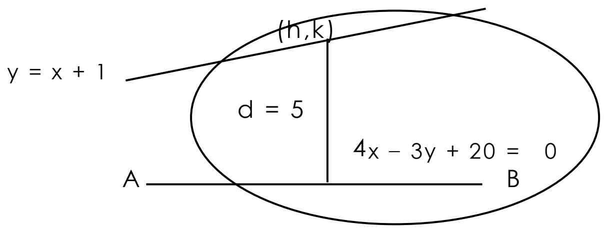
$$-24x + 15y = 180$$

$$24x - 15y + 180 = 0$$

$$8x - 5y + 60 = 0$$

02. Find points on the line $y = x + 1$ whose distance from $4x - 3y + 20 = 0$ is 5 units

SOLUTION



STEP 1 :

Since (h,k) lies on $y = x + 1 = 0$, it must satisfy the equation

$$\therefore k = h + 1 \qquad h - k = -1 \;\ldots\ldots (1)$$

STEP 2 :

$d = 5$

$$\left| \frac{4h - 3k + 20}{\sqrt{4^2 + 3^2}} \right| = 5$$

$$\frac{4h - 3k + 20}{5} = \pm 5$$

$$4h - 3k + 20 = \pm 25$$

$$4h - 3k + 20 = 25$$

$$4h - 3k = 5 \;\ldots. \quad (2)$$

Solving (1) & (2)

Eq (1) x 4

$$\begin{aligned} 4h - 3k &= 5 \\ 4h - 4k &= -4 \\ \hline k &= 9 \end{aligned}$$

subs in (1)

$$h - 9 = -1$$

$$h = 8$$

$(8 , 9)$

$$4h - 3k + 20 = -25$$

$$4h - 3k = -45 \;\ldots. \quad (3)$$

Solving (1) & (3)

Eq (1) x 4

$$\begin{aligned} 4h - 3k &= -45 \\ 4h - 4k &= -4 \\ \hline k &= -41 \end{aligned}$$

$$h + 41 = -1$$

$$h = -42$$

$(-42,-41)$

03. Find the angle subtended by the line segment PQ at the origin where $P \equiv (1,\sqrt{3})$ & $Q \equiv (\sqrt{3},1)$

SOLUTION

$$OP \ : \ m_1 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\sqrt{3} - 0}{1 - 0} = \sqrt{3}$$

$$OQ \ : \ m_2 = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 0}{\sqrt{3} - 0} = \frac{1}{\sqrt{3}}$$

let θ be the angle subtended by PQ at the origin
(θ is the angle between the lines OP & OQ)

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1.m_2}\right|$$

$$= \left|\frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} . \frac{1}{\sqrt{3}}}\right|$$

$$= \left|\frac{\frac{3-1}{\sqrt{3}}}{1+1}\right|$$

$$= \frac{1}{\sqrt{3}}$$

$$\theta = 30^\circ$$

(B) Attempt ANY **TWO OF** THE FOLLOWING (06)

01. find the acute angle θ satisfying : $4\sin^2\theta - 2(\sqrt{3} + 1)\sin\theta + \sqrt{3} = 0$

Q-2B

SOLUTION

$$4\sin^2\theta - 2\sqrt{3}\sin\theta - 2\sin\theta + \sqrt{3} = 0$$

$$2\sin\theta(2\sin\theta - \sqrt{3}) - 1(2\sin\theta - \sqrt{3}) = 0$$

$$(2\sin\theta - 1)(2\sin\theta - \sqrt{3}) = 0$$

$2\sin\theta - 1 = 0$ OR $2\sin\theta - \sqrt{3} = 0$

$\sin\theta = \frac{1}{2}$ $\qquad$ $\sin\theta = \frac{\sqrt{3}}{2}$

$\theta = 30^\circ$ $\qquad$ $\theta = 60^\circ$

02. Prove : $\dfrac{\operatorname{cosec}(90 - A) \,.\, \sin(180 - A) \,.\, \cot(360 - A)}{\sec(180 + A) \,.\, \tan(90 + A) \,.\, \sin(-A)} = 1$

SOLUTION

LHS

$$= \frac{\operatorname{cosec}(90^\circ - A) \,.\, \sin(180^\circ - A) \,.\, \cot(360^\circ - A)}{\sec(180^\circ + A) \,.\, \tan(90^\circ + A) \,.\, \sin(-A)}$$

$$= \frac{\sec A \,.\, \sin A .\, (-\cot A)}{(-\sec A) \,.\, (-\cot A) .\, (-\sin A)}$$

$= 1$

$=$ RHS

03. if $\sin A = {}^{4}/_{5}$, ${}^{\pi}/_{2} < A < \pi$ and $\cos B = {}^{5}/_{13}$, ${}^{3\pi}/_{2} < B < 2\pi$. find $\sin (A - B)$

SOLUTION

A lies in the II Quadrant

$\therefore$ sin A & cosec A are positive

$\sin A = \dfrac{4}{5}$

$\sin^2 A + \cos^2 A = 1$

$\dfrac{16}{25} + \cos^2 A = 1$

$\cos^2 A = 1 - \dfrac{16}{25}$

$\cos^2 A = \dfrac{9}{25}$

$\cos A = -\dfrac{3}{5}$

B lies in the IV Quadrant

$\therefore$ cos B & sec B are positive

$\cos B = \dfrac{5}{13}$;

$\sin^2 B + \cos^2 B = 1$

$\sin^2 B + \dfrac{25}{169} = 1$

$\sin^2 B = \dfrac{144}{169}$

$\sin B = -\dfrac{12}{13}$

Now

$\sin A = \dfrac{4}{5}$; $\cos A = -\dfrac{3}{5}$

$\sin B = \dfrac{-12}{13}$; $\cos B = \dfrac{5}{13}$

$\sin (A - B)$

$= \sin A \,.\, \cos B + \cos A \,.\, \sin B$

$= \dfrac{4}{5} \,.\, \dfrac{5}{13} \quad - \; -\dfrac{3}{5} \,.\, -\dfrac{12}{13}$

$= \dfrac{20}{65} - \dfrac{36}{65}$

$= - 16/65$

Q3. (A) Attempt ANY **ONE OF** THE FOLLOWING (04)

Q-3A

01. Four digit number (without repetition) is to be formed using digits 1 – 9 in all possible ways . Find how many of them are

a) greater than 4000 b) divisible by 2

4 digit numbers formed = ${}^{9}P_{4}$ = 3024

a) numbers greater than 4000

thousand's place can be filled by any of the digits 4 , 5 , ... , 9 in 6 ways

having done that , remaining three places can be filled by any 3 of the remaining 8 digits in ${}^{8}P_{3}$ ways

Hence By FUNDAMENTAL PRINICPLE OF MULTIPILCATION ,

Total numbers formed = $6 \times {}^{8}P_{3}$ = 2016

b) numbers divisible by 2

unit's place can be filled by any of the digits 2 , 4 , 6 , 8 in 4 ways

having done that , remaining three places can be filled by any 3 of the remaining 8 digits in ${}^{8}P_{3}$ ways

Hence By FUNDAMENTAL PRINICPLE OF MULTIPILCATION ,

Total numbers formed = $4 \times {}^{8}P_{3}$ = 1344

02. Find the number of all arrangement of the letters of the word 'TRIANGLE'

a) How many of them begin with T and end with E

b) In how many of them do the vowels occupy the odd places

c) in how many of them do the vowels occupy second , third and fourth places

SOLUTION

' TRIANGLE '

8 – LETTER WORD

VOWELS : - I , A , E CONSONANTS : - T , R , N , G , L

number of all arrangement of the letters of the word 'TRIANGLE' = ${}^{8}P_{8}$ = 8!

a) begin with T and end with E

First & the last place can be filled by letters 'T' and 'E' in 1 way

Having done that ,

Remaining 6 places can be filled by remaining 6 letters in ${}^{6}P_{6}$ = 6! Ways

By FUNDAMENTAL PRINCIPLE OF MULTIPLICATION ;

Total arrangements = 6! = 720

b) vowels occupy the odd places

the 3 vowels will occupy any 3 of the 4 odd places in ${}^{4}P_{3}$ ways

Having done that ,

Remaining 5 places can be filled by remaining 5 consonants in ${}^{5}P_{5}$ = 5! Ways

By FUNDAMENTAL PRINCIPLE OF MULTIPLICATION ;

Total arrangements = ${}^{4}P_{3}$ .5! = 2880

c) vowels occupy second , third and fourth places

the vowels will occupy second , third and fourth places in ${}^{3}P_{3}$ = 3! ways

Having done that ,

Remaining 5 places can be filled by remaining 5 consonants in ${}^{5}P_{5}$ = 5! Ways

By FUNDAMENTAL PRINCIPLE OF MULTIPLICATION ;

Total arrangements = 3! .5! = 720

(B) Attempt ANY **THREE OF** THE FOLLOWING (09)

Q-3B

01. Calculate the quartile deviation for the following data

CI	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	6	25	36	20	13

CI	f	cf	
0 – 10	6	6	
10 – 20	25	31	← Q1 CLASS
20 – 30	36	67	
30 – 40	20	87	← Q3 CLASS
40 – 50	13	100	

$$q_1 = \frac{N}{4} = \frac{100}{4} = 25$$

$$Q_1 = L_1 + \frac{q_1 - c}{f}(L_2 - L_1)$$

$$= 10 + \frac{25 - 6}{25}(20 - 10)$$

$$= 10 + \frac{19}{25}(10)$$

$$= 10 + 7.6$$

$$= 17.6$$

$$q_3 = \frac{3N}{4} = \frac{3\ 100}{4} = 75$$

$$Q_3 = L_1 + \frac{q_3 - c}{f}(L_2 - L_1)$$

$$= 30 + \frac{75 - 67}{20}(40 - 30)$$

$$= 30 + \frac{8}{20}(10)$$

$$= 30 + 4$$

$$= 34$$

$$QD = \frac{Q_3 - Q_1}{2} = \frac{34 - 17.6}{2} = \frac{16.4}{2} = 8.2$$

02. Calculate the mean deviation from the median . Also find the coefficient of MD

CI	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	25	25	18	7

STEP 1 : MEDIAN

CI	f	cf
0 – 10	5	5
10 – 20	25	30
20 – 30	25	55
30 – 40	18	73
40 – 50	7	80

$$m = \frac{N}{2} = \frac{80}{2} = 40$$

$$M = L1 + \frac{m - c}{f}(L_2 - L_1)$$

$$= 20 + \frac{40 - 30}{25}(30 - 20)$$

$$= 20 + \frac{10(10)}{25} = 25 \qquad M = 24$$

STEP 2 : MEAN DEVIATION FROM MEDIAN

X	f	|x – M |	f |x – M|
5	5	19	95
15	25	9	225
25	25	1	25
35	18	11	198
45	7	21	147
	80		690

$$MD_{(MEDIAN)} = \frac{\Sigma f|x-M|}{\Sigma f}$$

$$= \frac{690}{80}$$

$$= 8.625$$

$$\text{COEFF. OF MD} = \frac{MD(median)}{M}$$

$$= \frac{8.625}{80}$$

$$= 0.10$$

03. Solve : $\log_{\sqrt{3}} x + \log_3 x + \log_{\sqrt{27}} x = 11$

SOLUTION

$$\log_{\sqrt{3}} x + \log_3 x + \log_{\sqrt{27}} x = 11$$

$$\frac{\log x}{\log \sqrt{3}} + \frac{\log x}{\log 3} + \frac{\log x}{\log \sqrt{27}} = 11$$

$$\frac{\log x}{\log 3^{1/2}} + \frac{\log x}{\log 3} + \frac{\log x}{\log 3^{3/2}} = 11$$

$$\frac{\log x}{\frac{1}{2}\log 3} + \frac{\log x}{\log 3} + \frac{\log x}{\frac{3}{2}\log 3} = 11$$

$$\frac{2\log x}{\log 3} + \frac{\log x}{\log 3} + \frac{2\log x}{3\log 3} = 11$$

$$\frac{\log x}{\log 3}\left(2 + 1 + \frac{2}{3}\right) = 11$$

$$\frac{\log x}{\log 3}\left(\frac{6 + 3 + 2}{3}\right) = 11$$

$$\frac{\log x}{\log 3}\left(\frac{11}{3}\right) = 11$$

$$\frac{\log x}{\log 3} = 3$$

$$\log x = 3 \log 3$$

$$\log x = \log 3^3$$

$$x = 27$$

04. if $\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$, then show that $(x+y)^2 = 20xy$

SOLUTION :

$$\log\left(\frac{x-y}{4}\right) = \log\sqrt{x} + \log\sqrt{y}$$

$$\log\left(\frac{x-y}{4}\right) = \log\sqrt{x}.\sqrt{y}$$

$$\left(\frac{x-y}{4}\right) = \sqrt{x}.\sqrt{y}$$

Squaring both sides

$$\left(\frac{x-y}{4}\right)^2 = xy$$

$$\frac{x^2 - 2xy + y^2}{16} = xy$$

$$x^2 - 2xy + y^2 = 16xy$$

$$x^2 + y^2 = 18xy$$

Adding '2xy' on both sides

$$x^2 + 2xy + y^2 = 20xy$$

$$(x+y)^2 = 20xy$$ .... PROVED

Q4. (A) Attempt ANY **TWO OF** THE FOLLOWING **Q-4A** (06)

01. Find SD for the following data : 15 , 16 , 18 , 18 , 19 , 20 , 20 , 21 , 21, 22

x	$x - \bar{x}$	$(x - \bar{x})^2$
15	−4	16
16	−3	9
18	−1	1
18	−1	1
19	0	0
20	1	1
20	1	1
21	2	4
21	2	4
22	3	9
190		46

$$\bar{x} = \frac{\Sigma x}{n} = \frac{190}{10} = 19$$

$$\sigma = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$= \sqrt{\frac{46}{10}}$$

$$= \sqrt{4.6}$$

taking log on both sides

$$\log \sigma = \frac{1}{2}(\log 4.6)$$

$$= \frac{1}{2}(0.6628)$$

$$= 0.3314$$

$$\sigma = AL(0.3314)$$

$$= 2.145$$

02. Find Bowley's coefficient of skewness for the following data

11 , 8 , 3 , 10 , 6 , 10 , 1

SOLUTION

obs no.	1	2	3	4	5	6	7
value	1	3	6	8	10	10	11

STEP 1 :

$Q_1 = $ value of the $\frac{N+1}{4}^{th}$ observation

$=$ value of 2^{nd} observation $= 3$

$Q_2 = $ value of the $\frac{N+1}{2}^{nd}$ observation

$=$ value of 4^{th} observation $= 8$

$Q_3 = $ value of the $3\ \frac{N+1}{4}^{th}$ observation

$=$ value of 6^{th} observation $= 10$

STEP 2 :

$Q_3 - Q_2 = 2$

$Q_2 - Q_1 = 5$

$$SK_B = \frac{(Q_3 - Q_2) - (Q_2 - Q_1)}{(Q_3 - Q_2) + (Q_2 - Q_1)} = \frac{2-5}{2+5} = \frac{-3}{7} = -0.43$$

03. For a moderately skewed distribution

Mean = 200 ; median = 198.4 , SD = 16 . Find mode and the Karl Pearson's coefficient of skewness (SKp)

STEP 1 : MODE

Mean – mode = 3(mean - median)

200 – mode = 3(200 – 198.4)

200 – mode = 3(1.6)

200 – mode = 4.8

mode = 200 – 4.8

= 195.2

STEP 2 :KARL PERASON COEFF. OF SKEWNESS

$$Sk_P = \frac{3(\text{Mean} - \text{Median})}{\sigma}$$

$$= \frac{3(200 - 198.4)}{16}$$

$$= \frac{3(1\cancel{.6})}{\cancel{16}\ 10} = \frac{3}{10} = 0.3$$

(B) Attempt ANY **ONE OF** THE FOLLOWING (03)

01. For the following data find the age above which we have the oldest 20% of persons

Age	Below 35	35 – 50	50 – 65	65 – 80	Above 80
Frequency	8	22	25	17	8

SOLUTION

Q-4B

CI	f	cf
Below 35	8	8
35 – 50	22	30
50 – 65	25	55
65 – 80	17	72
above 80	8	80

$$d_8 = \frac{8N}{10} = \frac{8(80)}{10} = 64$$

$$D_8 = L_1 + \frac{d_8 - c}{f}(L_2 - L_1)$$

$$= 65 + \frac{64 - 55}{17}(80 - 65)$$

$$= 65 + \frac{9(15)}{17}$$

$$= 65 + 7.94$$

$$= 72.94 \text{ yrs}$$

02. following is the distribution of age of 500 workers , find the percentage of workers whose age is more than 45 years

Age	20 – 30	30 – 40	40 – 50	50 – 60
No of workers	80	160	180	80

SOLUTION :

CI	f	cf
20 – 30	80	80
30 – 40	160	240
40 – 50	180	420 ←
50 – 60	80	500

let age of nth worker be 45 .This worker is in class 40 – 50

$$45 = 40 + \frac{n - 240}{180}(50 - 40)$$

$$5 = \frac{n - 240}{180} . (10)$$

$$5 = \frac{n - 240}{18}$$

$$90 = n - 240 \quad \therefore n = 330$$

$\therefore$ age of 330^{th} worker is 45

$\therefore$ No. of workers whose age is more than 45 years $= 500 - 330 = 170$

$\therefore$ Percentage of workers with age more than 45 years $= \frac{170}{500} \times 100 = 34\%$

(C) Attempt ANY **ONE** **OF** THE FOLLOWING (03)

Q-4C

01. Find n , ${}^{n}P_3 : {}^{n}P_6 = 1 : 210$

SOLUTION

$$\frac{{}^{n}P_3}{{}^{n}P_6} = \frac{1}{210}$$

$$\frac{\frac{n!}{(n-3)!}}{\frac{n!}{(n-6)!}} = \frac{1}{210}$$

$$\frac{(n-6)!}{(n-3)!} = \frac{1}{210}$$

$$\frac{(n-6)!}{(n-3)(n-4)(n-5)(n-6)!} = \frac{1}{210}$$

$$\frac{1}{(n-3)(n-4)(n-5)} = \frac{1}{210}$$

$$(n-3)(n-4)(n-5) = 210$$

$$(n-3)(n-4)(n-5) = 7.6.5$$

2	210
3	105
5	35
7	7
	1

(DESCENDING ORDER)

On Comparing

$n - 3 = 7$

$n = 10$

02. the first four moments about 4 are 1 , 4 , 10 , 46 . Find Personian's coefficients of kurtosis

SOLUTION :

$A = 4 ; \mu_1(a) = 1 ; \mu_2(a) = 4 ; \mu_3(a) = 10 ; \mu_4(a) = 46$

$$\mu_2 = \mu_2(a) - \mu_1(a)^2$$
$$= 4 - (1)^2$$
$$= 3$$

$$\mu_3 = \mu_3(a) - 3\,\mu_1(a) . \mu_2(a) + 2\,\mu_1(a)^3$$

NOT REQUIRED

$$\mu_4 = \mu_4(a) - 4\,\mu_1(a) . \mu_3(a) + 6\,\mu_2(a) \cdot \mu_1(a)^2 - 3\,\mu_1(a)^4$$
$$= 46 - 4(1)(10) + 6(4)(1)^2 - 3(1)^4$$
$$= 46 - 40 + 24 - 3$$
$$= 70 - 43$$
$$= 27$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{27}{(3)^2} = 3$$

$$\gamma_2 = \beta_2 - 3 = 3 - 3$$
$$= 0$$

COMMENT

DISTRIBUTION IS MESOKURTIC